Mechanics of machinery

CAMS AND FOLLOWERS

INTRODUCTION

✓A cam is a mechanical member used to impart desired motion

to a follower by direct contact

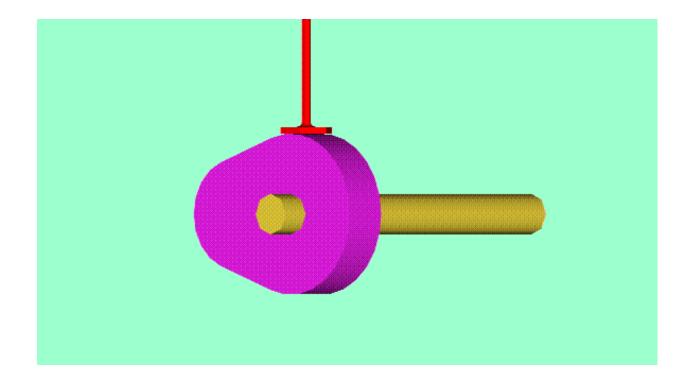
✓The cam may be rotating or reciprocating whereas the follower

may be rotating, reciprocating or oscillating

✓Complicated output motions which are otherwise difficult to

achieve can easily be produced with the help of cams

CAM AND FOLLOWER



TYPES OF CAMS

- a) According to shape:
- 1. Wedge and Flat Cams
- 2. Radial or Disc Cams
- 3. Spiral Cams
- 4. Cylindrical Cams
- 5. Conjugate Cams
- 6. Globoidal Cams
- 7. Spherical Cams

1. Wedge and Flat Cams

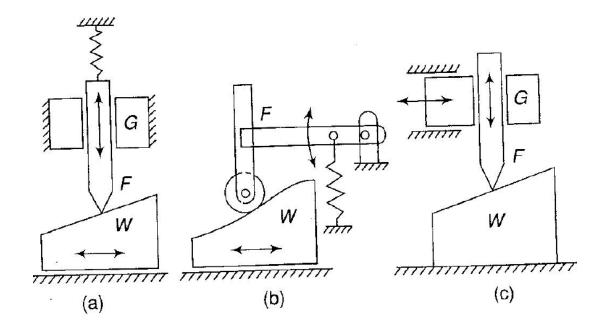
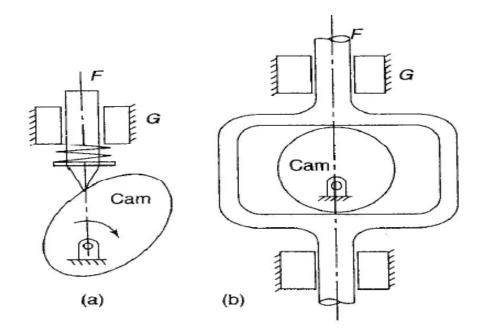


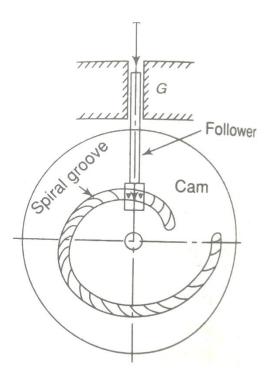
Fig (a)- Wedge W translates, follower F translates Fig(b)- Wedge W translates, follower F oscillates Fig(c)- Wedge W is stationary, guide G causes the relative motion of follower and cam

2. Radial or Disc Cams

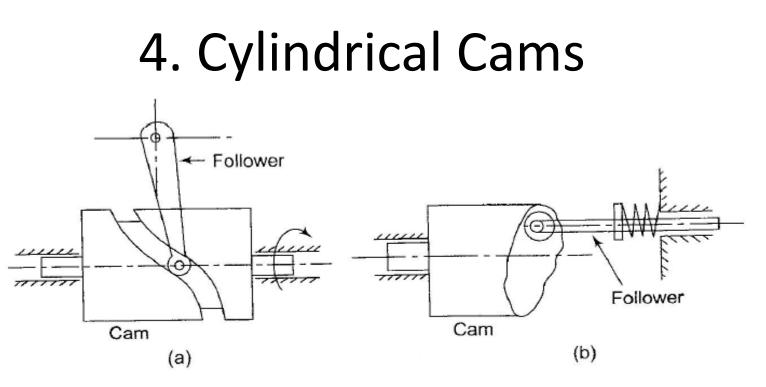


•Follower moves radially from the centre of rotation of the cam

3. Spiral Cams



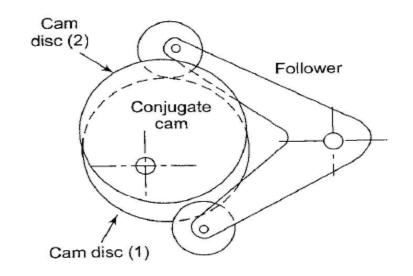
It is a face cam in which a groove is cut in the form of a spiral
The velocity of the follower is proportional to the radial distance of the groove from the axis of the cam



The cylinder has a circumferential contour cut in the surface, and rotates about its axis. The follower motion can be of two types:-

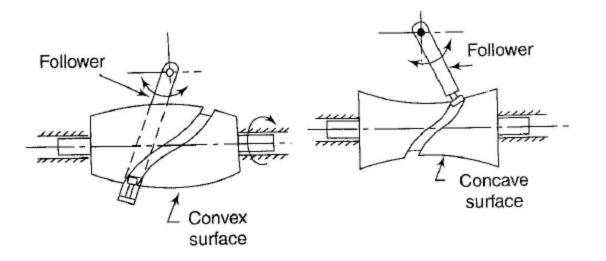
Fig (a)- A groove is cut on the surface of the cam and a follower has a constrained oscillating motion Fig (b)- A spring loaded follower translates along or parallel to the axis of the rotating cylinder

5. Conjugate cams



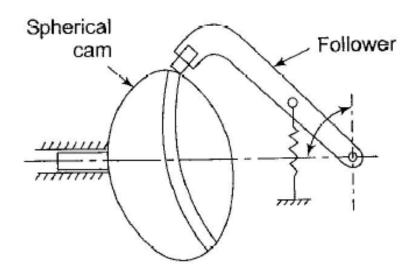
It is a double disc cam, the two discs being keyed together and are in constant touch with the two rollers of a follower

6. Globoidal Cams



A globoidal cam can have two types of surfaces , convex or concave. A circumferential contour is cut on the surface of the rotation of the cam to impart motion to the follower which has an oscillatory motion

7. Spherical cams



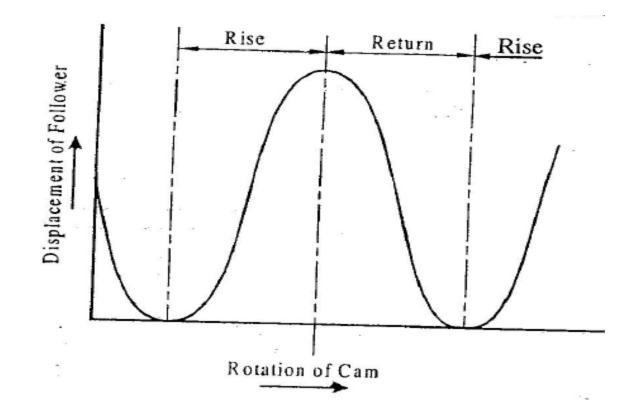
In a spherical cam, the follower oscillates about an axis perpendicular to the axis of rotation of the cam
A spherical cam is in the form of a spherical surface which transmits motion to the follower.

b) Classification of cams according to follower movement

- 1. Rise- Return- Rise (R-R-R)
- 2. Dwell- Rise- Return- Dwell (D-R-R-D)
- 3. Dwell- Rise- Dwell- Return- Dwell (D-R-D-R-D)

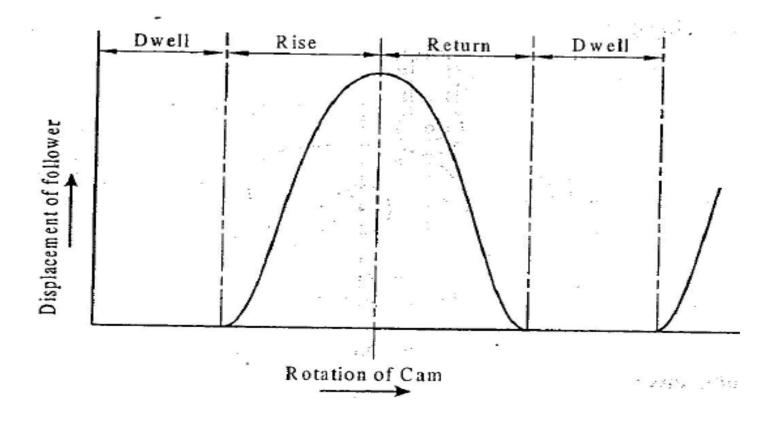
R-R-R (Rise- Return- Rise)

In this there is alternate rise and return of the follower with no periods of dwells



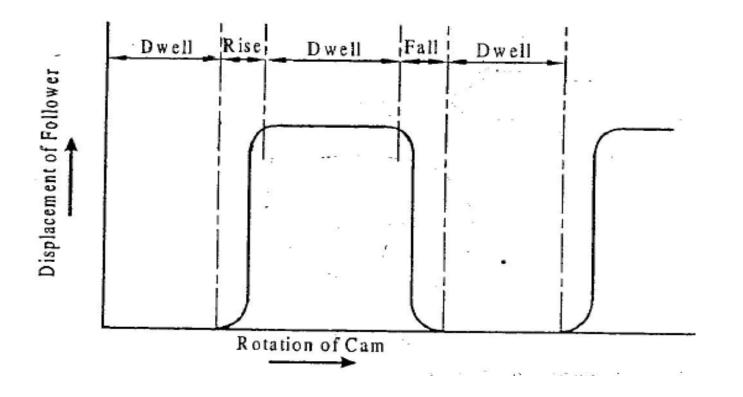
D-R-R-D (Dwell- Rise-Return- Dwell)

In such a type of cam, there is rise and return of the follower after a dwell.



Dwell- Rise- Dwell- Return- Dwell (D-R-D-R-D)

The dwelling of the cam is followed by rise and dwell and subsequently by return and dwell

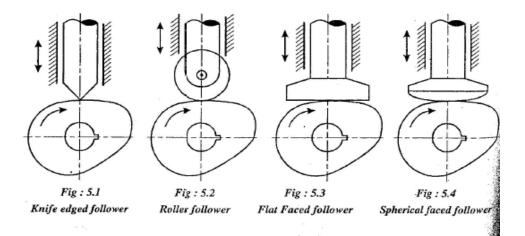


Types of Followers

- Followers are classified according to :-
- a) Shape
- b) Movement and
- c) Location of line of movement

a) According to shape

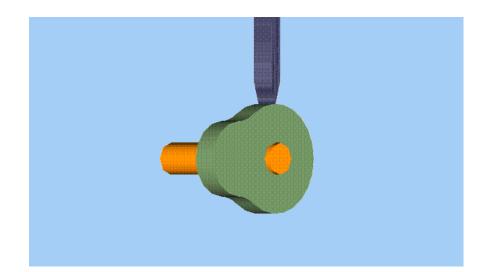
- 1. Knife- edge follower
- 2. Roller follower
- 3. Mushroom Follower (flat or spherical faced follower)

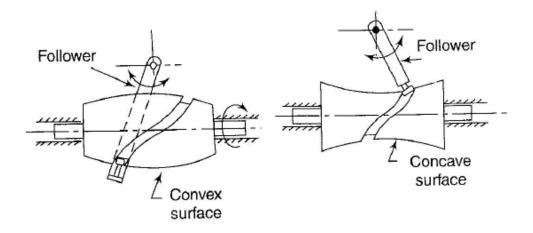




b) According to movement

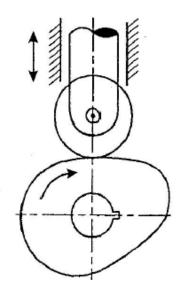
- 1. Reciprocating Follower
- 2. Oscillating follower



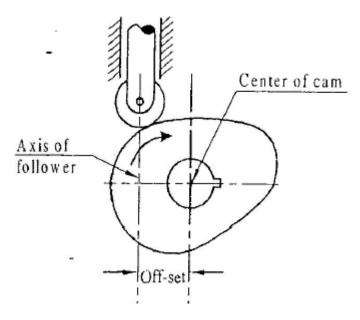


c) According to the location of line of movement

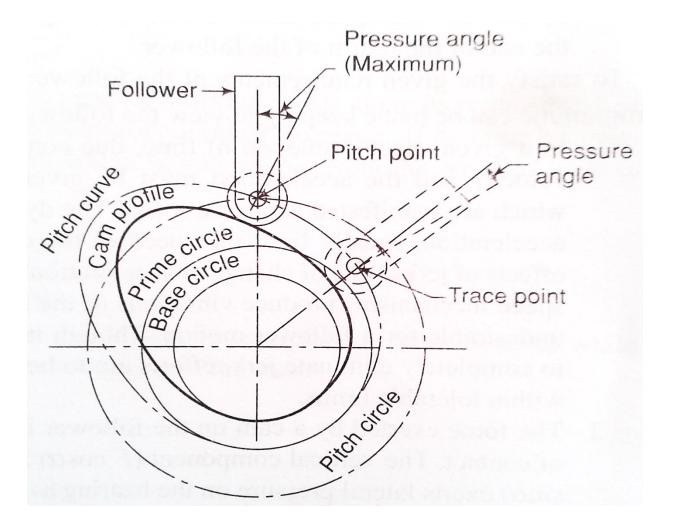
1. Radial follower



2. Offset follower



Terminology



Cam profile- It is the actual working surface contour of the cam

Base circle- It is the smallest circle drawn to the cam profile from

the centre of rotation of the cam

Trace point- It is the point on the follower located at the knife

edge in knife edge follower and the centre of roller in the roller

follower or the centre of spherical face in mushroom spherical

follower

Pitch curve- it is the curve generated by the trace point as the

follower moves relative to the cam

Lift or stroke- It is the maximum displacement of the follower

from its lowest position to the topmost position

Pressure angle- It is the angle between the normal to the pitch

curve and the instantaneous direction of the follower motion

Pitch point- point on the pitch curve where the pressure angle is

maximum

Dwell- It is the period during which the follower is at rest

Pitch circle- It is the circle from the cam centre through the pitch point

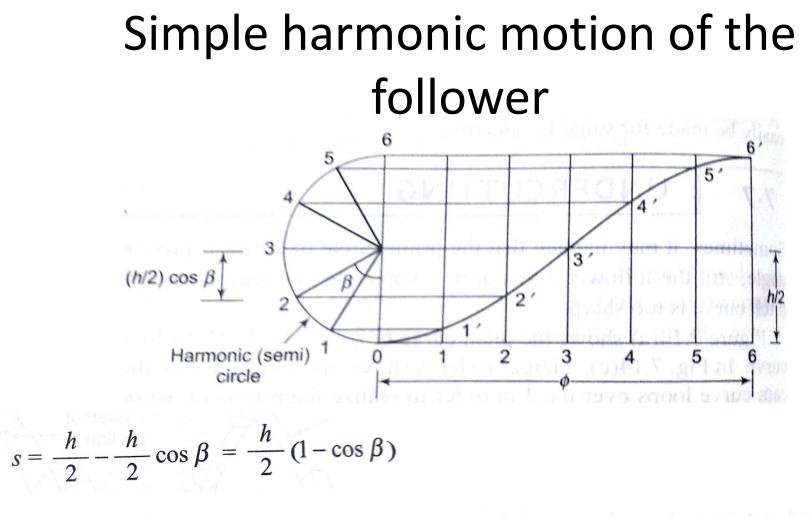
Prime circle- It is the smallest circle drawn to the pitch curve

from the centre of rotation of the cam

Cam angle- It is the angle of rotation of the cam for a definite

displacement of the follower.

Follower Motion



 $\beta = \pi \frac{\theta}{\varphi}$ $s = \frac{h}{2} \left(1 - \cos \frac{\pi \theta}{\varphi} \right)$

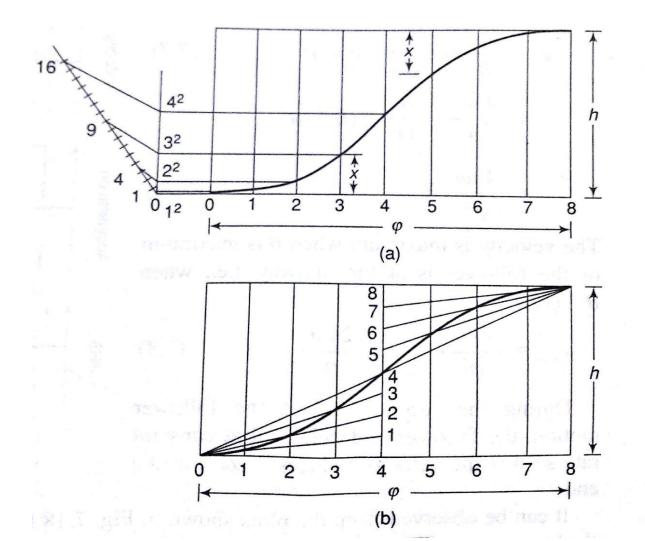
 ω = Angular velocity of the cam

$$s = \frac{h}{2} \left(1 - \cos \frac{\pi \omega t}{\varphi} \right)$$

A - at

$$v = \frac{ds}{dt} = \frac{h}{2} \frac{\pi\omega}{\varphi} \sin \frac{\pi\omega t}{\varphi}$$
$$= \frac{h}{2} \frac{\pi\omega}{\varphi} \sin \frac{\pi\theta}{\varphi}$$
$$v_{\text{max}} = \frac{h}{2} \frac{\pi\omega}{\varphi} \sin \frac{\pi\theta}{\varphi}$$
$$t^{\text{max}} = \frac{h}{2} \frac{\pi\omega}{\varphi} at \theta = \frac{\varphi}{2}$$
$$f = \frac{dv}{dt} = \frac{h}{2} \left(\frac{\pi\omega}{\varphi}\right)^2 \cos \frac{\pi\omega t}{\varphi}$$
$$= \frac{h}{2} \left(\frac{\pi\omega}{\varphi}\right)^2 \cos \frac{\pi\theta}{\varphi}$$
$$f_{\text{max}} = \frac{h}{2} \left(\frac{\pi\omega}{\varphi}\right)^2 at \theta = 0$$

Constant acceleration and deceleration of the follower



$$s = v_0 t + \frac{1}{2} ft^2$$

where v_o is the initial velocity at the start of the motion (rise or fall) and is zero in this case.

$$\therefore \qquad S = \frac{1}{2} ft^2$$
or
$$f = \frac{2s}{t^2} = \text{constant}$$

$$\int \frac{1}{2} ft^2 = \frac{1}{2} ft^2$$

$$\int \frac{1}{2} \int \frac{1}{2} ft^2 = \frac{1}{2} \int \frac{1}{2}$$

As f is constant during the accelerating period, considering the follower at the midway,

$$s = \frac{h}{2} \text{ and } t = \frac{\varphi/2}{\omega}$$

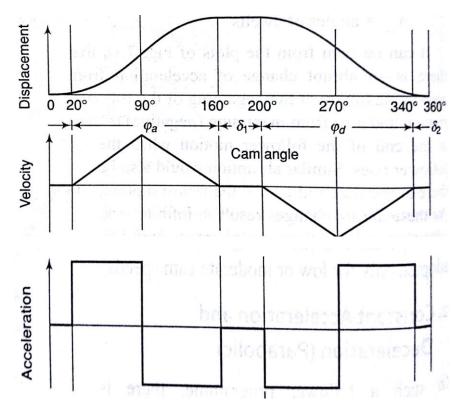
$$f = \frac{2h/2}{\varphi^2/4\omega^2} = \frac{4h\omega^2}{\varphi^2}$$

The velocity is linear during the period and is given by

$$w = \frac{ds}{dt} = \frac{1}{2} \times 2ft = ft$$
$$= \frac{4h\omega^2}{\varphi^2} \frac{\theta}{\omega} \quad (\theta = \omega t)$$
$$= \frac{4h\omega}{\varphi^2} \theta$$

The velocity is maximum when θ is maximum or the follower is at the midway, i.e., when $\theta = \varphi/2$.

$$v_{\rm max} = \frac{4h\omega}{\varphi^2} \frac{\varphi}{2} = \frac{2h\omega}{\varphi}$$



Constant Velocity motion of the follower

$$s = h \frac{\theta}{\varphi} = h \frac{\omega t}{\varphi}$$
$$v = \frac{ds}{dt} = \frac{h\omega}{\varphi} = \text{constant}$$
$$f = \frac{dv}{dt} = 0$$

Horizon velocity
$$\varphi_a$$
 φ_{d} φ_{d}

Cycloidal motion of the follower

$$s = \frac{h}{\pi} \left(\frac{\pi\theta}{\varphi} - \frac{1}{2} \sin \frac{2\pi\theta}{\varphi} \right)$$

$$v = \frac{ds}{dt} = \frac{ds}{d\theta} \frac{d\theta}{dt}$$

$$= \left[\frac{h}{\varphi} - \frac{h}{2\pi} \frac{2\pi}{\varphi} \cos \frac{2\pi\theta}{\varphi} \right] \omega$$

$$= \frac{h\omega}{\varphi} - \frac{h\omega}{\varphi} \cos \frac{2\pi\theta}{\varphi}$$

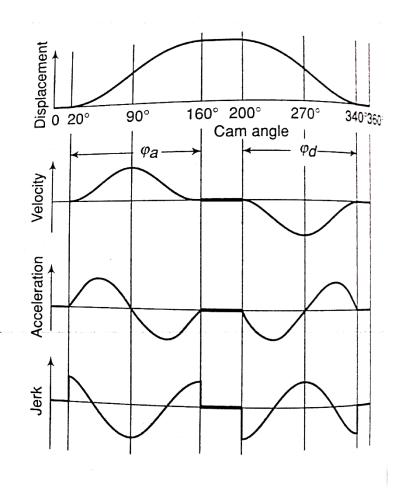
$$= \frac{h\omega}{\varphi} \left(1 - \cos \frac{2\pi\theta}{\varphi} \right)$$

$$v_{\text{max}} = \frac{2h\omega}{\varphi} \text{ at } \theta = \frac{\varphi}{2}$$

$$f = \frac{dv}{dt} = \frac{dv}{d\theta} \frac{d\theta}{dt}$$

$$= \left[\frac{h\omega}{\varphi} \frac{2\pi}{\varphi} \sin \frac{2\pi\theta}{\varphi} \right] \omega$$

$$= \frac{2h\pi\omega^2}{\varphi^2} \sin \frac{2\pi\theta}{\varphi}$$



Tangent cam (with roller follower)

A tangent cam is symmetrical about the centre line. It has straight flanks (such as AK in figure) with a circular nose. The centre of cam is at O and that of nose is at Q.

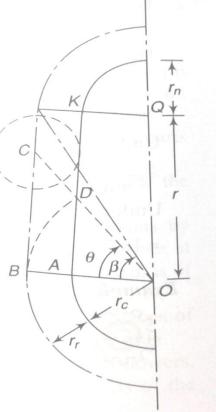
- r_c = least radius of cam
- $r_n = radius of nose$

r

θ

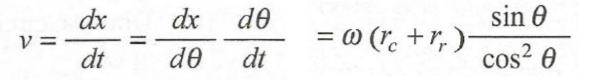
β

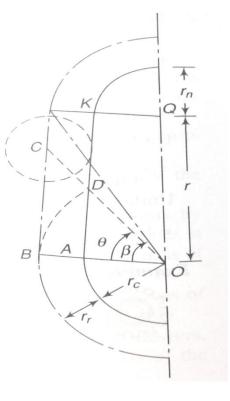
- $r_r = radius of roller$
 - = distance between the cam and nose centers
 - = angle turned by cam
 - = angle turned by the cam when the roller leaves the flank



Roller on flank:

$$x = OC - OD = OC - OB$$
$$x = (r_c + r_r) \left(\frac{1}{\cos \theta} - 1\right)$$





Let β be the angle turned by the cam when the roller leaves the flank, then

$$v_{\max} = \omega (r_c + r_r) \frac{\sin \beta}{\cos^2 \beta} \text{ and } v_{\min} = 0 \text{ at } \theta = 0$$

$$f = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt} = \frac{\omega^2 (r_c + r_r) (2 - \cos^2 \theta)}{\cos^3 \theta}$$

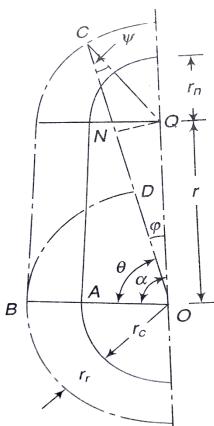
$$f_{\min} = \omega^2 (r_c + r_r) \quad \text{when } \theta = 0^\circ$$

Roller on the nose:

$$x = OC - OD$$

= $(ON + NC) - OB$
= $OQ \cos \varphi + CQ \cos \psi - OB$
= $OQ \cos \varphi + CQ\sqrt{1 - \sin^2 \psi} - OB$
= $OQ \cos \varphi + CQ\sqrt{1 - \frac{(NQ)^2}{(CQ)^2}} - OB$
= $r \cos (\alpha - \theta) + \sqrt{l^2 - r^2 \sin^2 (\alpha - \theta)} - n$
 $l = CQ = r_n + r_r$
 $n = OB = r_c + r_r$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \frac{d\theta}{dt}$$
$$= \omega r \left[\sin(\alpha - \theta) + \frac{r \sin 2(\alpha - \theta)}{2\sqrt{(l^2 - r^2 \sin^2(\alpha - \theta))}} \right]$$
$$f = \frac{dv}{dt} = \frac{dv}{d\theta} \frac{d\theta}{dt}$$
$$= \omega^2 r \left[-\cos(\alpha - \theta) - \frac{r^3 \sin^2 2(\alpha - \theta)}{4[l^2 - r^2 \sin^2(\alpha - \theta)]^{3/2}} - \frac{r \cos 2(\alpha - \theta)}{\sqrt{l^2 - r^2 \sin^2(\alpha - \theta)}} \right]$$

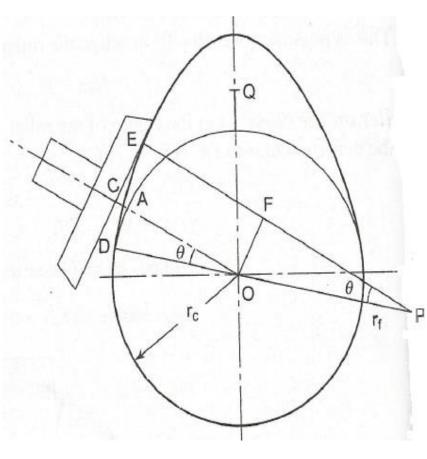


θ

Circular arc convex cam (with flat face follower)

• made of three arcs of different radii

• In such cams acceleration changes abruptly at the blending points due to instantaneous change in the radius of curvature.



Follower touching circular flank:

P is the centre of circular arc of the flank r_f is the radius of the circular flank, PD=PE

$$\begin{aligned} x &= OC - OA = EF - OA = (PE - PF) - OA \\ &= PE - OP \cos \theta - OA \\ &= PE - (PD - OD) \cos \theta - OA \\ x &= r_f - (r_f - r_c) \cos \theta - r_c \\ &= r_f - r_f \cos \theta + r_c \cos \theta - r_c \\ &= r_f (1 - \cos \theta) - r_c (1 - \cos \theta) \\ &= (r_f - r_c) (1 - \cos \theta) \end{aligned}$$

$$DA$$
 D θ T_{C} P θ T_{T} P

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \frac{d\theta}{dt} = \omega (r_f - r_c) \sin \theta$$

$$f = \frac{dv}{dt} = \frac{dv}{d\theta} \frac{d\theta}{dt} = \left[(r_f - r_c) \cos \theta \right] \omega = \omega^2 (r_f - r_c) \cos \theta$$

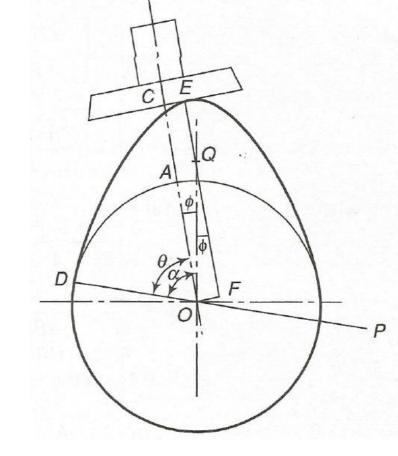
$$f_{\text{max}} = \omega^2 (r_f - r_c) \qquad \text{at the beginning when } \theta = 0$$

$$f_{\text{min}} = \omega^2 (r_f - r_c) \cos \beta$$

Follower touching nose:

$$x = OC - OA = EF - OA$$
$$= QE + QF - OA$$
$$= QE + OQ \cos\varphi - OA$$
$$= r_n + r \cos(\alpha - \theta) - r_c$$
$$= r_n - r_c + r \cos(\alpha - \theta)$$
$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \frac{d\theta}{dt}$$
$$= \omega r \sin(\alpha - \theta)$$

 $f = \frac{dv}{dt} = \frac{dv}{d\theta} \frac{d\theta}{dt} = \omega r \cos(\alpha - \theta)(-1)\omega = -\omega^2 r \cos(\alpha - \theta)$



Circular arc convex cam (with roller follower)

Follower on the flank:

$$x = OC - OA = FC - FO - OB$$

$$x = (r_f + r_r) \cos \varphi - (r_f + r_c) \cos \theta - (r_c + r_r)$$

$$\cos \varphi = \sqrt{1 - \left(\frac{A \sin \theta}{B}\right)^2} = \frac{1}{B} \sqrt{B^2 - A^2 \sin^2 \theta}$$
where $A = r_f - r_c$ and $B = r_f + r_r$

$$x = \sqrt{B^2 - A^2 \sin^2 \theta} - A \cos \theta - (r_c + r_r)$$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \frac{d\theta}{dt} = \omega A \left[\sin \theta - \frac{A \sin 2\theta}{2\sqrt{B^2 - A^2 \sin^2 \theta}} \right]$$

$$f = \frac{dv}{dt} = \frac{dv}{d\theta} \frac{d\theta}{dt} = \omega^2 A \left[\cos \theta - \frac{A \cos 2\theta}{\sqrt{B^2 - A^2 \sin^2 \theta}} - \frac{A^3 \sin 2\theta}{4(B^2 - A^2 \sin^2 \theta)^{3/2}} \right]$$